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Open string attractors

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Abstract: We present a simple example of a supersymmetric attractor mechanism in the purely open string context of D-branes embedded in curved space-time. Our example involves a class of D3-branes embedded in the 2-charge D1-D5 background of type IIB whose worldvolume contains a 2-sphere. Turning on worldvolume fluxes, these branes carry induced (p, q) string charges. Supersymmetric configurations display a flow of the open string moduli towards an attractor solution independent of their asymptotics. The equations governing this mechanism closely resemble the attractor flow equations for supersymmetric black holes in closed string theory. The BPS equations take the form of a gradient flow and describe worldvolume solitons interpolating between an AdS_2 geometry where the two-sphere has collapsed, and an attractor solution with $AdS_2 \times S^2$ geometry. In these limiting solutions, the preserved supersymmetry is enhanced from 4 to 8 supercharges. We also discuss the interpretation of our solutions as intersecting brane configurations placed in the D1-D5 background, as well as the S-duality transformation to the F1-NS5 background.

Keywords: D-branes, Supersymmetry and Duality, Black Holes in String Theory.

Contents

1. Introduction

Extremal black holes in string theory have the property that scalar moduli fields are drawn to fixed values at the horizon, which are determined by the charges carried by the black hole. This property, which goes under the name of the attractor mechanism, has played an important role in the understanding of black holes in string theory. It was first discovered in the context of supersymmetric black holes in $N = 2$ theories [1−3] and more recently, has played a crucial role in the formulation of the OSV-conjecture [4] and has been shown to apply to nonsupersymmetric black holes as well [5, 6]. The physics underlying the mechanism is closely tied to the microscopic entropy carried by the black hole: since the size of the horizon depends on the moduli, the latter should approach values determined by the black hole charges and independent of their continuous asymptotic values.

General open-closed string duality considerations lead one to expect that an attractor mechanism should exist for open string moduli as well. In the low-energy limit, open string dynamics is described by D-brane effective actions consisting of Born-Infeld and Wess-Zumino terms, and closer inspection shows that an attractor mechanism could occur in situations where both background and worldvolume gauge fields are turned on. For example, consider a background containing p-branes producing a RR electric potential C and a $p+2$ brane probe wrapping a transverse 2-cycle with worldvolume magnetic field F on this cycle. F is quantized and represents a lower D-brane charge. The term $\int F \wedge C$ in the worldvolume action then represents a potential term for the scalar moduli that describe the D-brane embedding, which is determined by the background and worldvolume charges and which vanishes far away from the branes in the background. It is therefore reasonable to expect that this interactions fixes a combination of open string moduli in terms of the worldvolume and background charges in the vicinity of the branes in the background. This example can then be dualized to more general situations. Such a mechanism is of course closely related to the stabilization of open string moduli in situations with background and worldvolume fluxes, which was explored in [7], and ultimately goes back to the observation of flux stabilization of D-branes [8]. We should stress that, although the appearance of an open string attractor mechanism seems plausible from the explicit form of the interactions, it is not a priori clear whether there is an underlying explanation in terms of an entropy contained in the open degrees of freedom. The open string attractor mechanism is also expected to play an important role in the open string version of the OSV conjecture proposed in [9], which could shed light on a possible entropic interpretation.

In this work, we will describe in detail an explicit example of such an open string attractor mechanism. We will consider here only the supersymmetric version, although the above considerations suggest that, just like in the closed string case, the mechanism is not restricted to the supersymmetric context. Our main example will display a similarity to the supersymmetric attractor mechanism in the closed string context that we find rather striking and deserving of a better explanation than we will be able to give at present. Our example involves a class of D3-brane probes in the 'D1-D5 system' (for a review, see [10]): type IIB compactified on a fourfold $\mathcal{M}$, with D5-branes wrapping $\mathcal{M}$ and coinciding with D1-branes in a noncompact direction, forming a six-dimensional black string. We consider D3-brane probes where the worldvolume geometry contains a two sphere whose radius is allowed to vary over a 1+1 dimensional base. Such 'spherically symmetric' configurations preserve an SU(2) subgroup of the target space isometry group. Turning on worldvolume electric and magnetic fluxes along the base and the S^2 fibre respectively, the configurations carry fundamental and D-string charges and can be seen as 'fuzzy' (p, q) string expanded to form a D3 brane through a form of the Myers effect. We derive a BPS bound on the energy for D-brane configurations of this type, and show that the BPS-equations take the form of a gradient flow.

In the near-horizon limit, the background geometry becomes $AdS_3 \times S^3$, and the BPS flow equations take a form that is remarkably similar to the attractor flow equations for supersymmetric black holes. Fixed points of the flows correspond to extrema of a real and positive function Z. General flows represent worldvolume solitons which interpolate between a repulsive fixed point (a maximum of Z) at radial infinity, corresponding to an AdS_2 worldvolume geometry where the S^2 has collapsed to zero size, and the attractive fixed point (the minimum of Z) with $AdS_2 \times S^2$ worldvolume geometry near $r = 0$. The generic solution preserves 4 of the 'Poincaré' supersymmetries which extend to the full asymptotically flat geometry, while for the fixed point solutions the supersymmetry is enhanced to 8 supercharges. The solution at the attractive fixed point can also be obtained by extremizing an effective potential or 'entropy' function [11], whose physical meaning is less clear in this setting. In the open string metric, the radii of the AdS_2 and S^2 factors become equal.

We will also investigate how our solutions extend to the full asymptotically flat background. This clarifies their interpretation as intersecting brane configurations placed in the D1-D5 background. The general solution corresponds to a D3-brane transverse to the D1-D5 string in the background, with a (p, q) string 'spike' running between the two. The transverse distance between the D3-brane and the D1-D5 string becomes the asymptotic value of a modulus in the near-horizon limit. The near horizon solutions with enhanced supersymmetry and AdS_2 or $AdS_2 \times S^2$ geometry correspond to the limiting cases where the transverse distance is taken to infinity or zero respectively. We also discuss how our solutions transform under S-duality to the F1-NS5 background.

Let us also comment on related D-brane solutions that have appeared in the literature. The $AdS_2 \times S^2$ solution in the D1-D5 system was studied in [12 – 14]. The S-dual solution in the F1-NS5 background was first introduced in [15] and has been studied extensively in the literature, as sampling of which is [16]. Similar D-brane solutions also exist in the Klebanov-Strassler [17] and Maldacena-Nunez [18] backgrounds in the form of a (p, q) string expanding to form a D3-brane wrapping an S^2 within the S^3 [19]. Our solutions for general flows are related to the 'baryon vertex' solutions and their generalizations [20 – 24].

This paper is organized as follows. In section 2, we introduce our brane configurations and derive the BPS equations in both the asymptotically flat and near-horizon backgrounds. In section 3, we study the attractor flow equations in the near-horizon geometry and point out several analogies with supersymmetric black hole attractors. Section 4 discusses the extension of the brane solutions to the asymptotically flat spacetime and clarifies their interpretation. In section 5 we discuss the supersymmetries preserved by our solutions, which provides an alternative derivation of the BPS equations. We obtain the S-dual brane solutions in the F1-NS5 background in section 6 and end with a discussion in section 7. Appendix A clarifies the interpretation of our brane configurations as (p, q) strings expanded to a fuzzy D3-brane through a form of the Myers effect, while appendix B gives a derivation of the Killing spinors of the background in our conventions.

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2. Spherical D3-branes in the D1-D5 background

In this section we will set the stage for what is to be our main example of an open string attractor. We will consider a class of BPS D3-branes in the D1-D5 background, whose worldvolume geometry has an S^2-fibre, and derive the equations for their embedding into the background geometry from a bound on the energy. Of course, the resulting system can also be derived from supersymmetry preservation, which we will do in section 5. We will treat the full asymptotically flat background and the near-horizon limit simultaneously in this section, providing a more detailed discussion for each case in later sections.

2.1 Background

We start by displaying our conventions for the D1-D5 background geometry. We consider type IIB on $\mathcal{M}$ ($\mathcal{M}$ being either K_3 or T^4), with Q_5 D5-branes wrapped on $\mathcal{M}$ and Q_1 D1-branes, running parallel along a noncompact direction x. We choose spherical polar coordinates for the remaining transverse noncompact directions. The string metric, dilaton and and RR three-form field strength are

$$\begin{aligned} ds^2 &= (H_1H_5)^{-1/2}(-dt^2+dx^2)+(H_1H_5)^{1/2}(dr^2+r^2d\Omega_3^2)+\left(\frac{H_1}{H_5}\right)^{1/2}ds^2_{\mathcal{M}} \\ e^{-\Phi} &= \frac{1}{g}\left(\frac{H_5}{H_1}\right)^{1/2} \\ F^{(3)} &= \frac{2r_1^2}{gr^3H_1^2}dt\wedge dx\wedge dr+\frac{2r_5^2}{g}\sin^2\psi\sin\theta d\psi\wedge d\theta\wedge d\phi \end{aligned} \tag{2.1}$$

where $ds^2_{\mathcal{M}}$ is the Ricci-flat metric on $\mathcal{M}$ and $d\Omega_3^2$ is the metric on a unit S^3. We choose angular coordinates ψ,θ,ϕ on the S^3:

$$d\Omega_3^2 = d\psi^2+\sin^2\psi(d\theta^2+\sin^2\theta d\phi^2). \tag{2.2}$$

where $\psi,\theta\in[0,\pi],\ \phi\in[0,2\pi]$. Note that the surfaces of constant ψ are 2-spheres of radius $\sin\psi$. The harmonic functions appearing in (2.1) are

$$H_{1,5}=a+\frac{r_{1,5}^2}{r^2};\qquad r_1=\frac{4\pi^2\alpha'}{\sqrt{V_M}}\sqrt{gQ_1\alpha'},\qquad r_5=\sqrt{gQ_5\alpha'}$$

where $a=1$ describes the asymptotically flat geometry while taking $a=0$ gives the near-horizon $AdS_3\times S^3\times\mathcal{M}$ geometry in the Poincaré patch. We will work in the following gauge for the RR two-form:

$$C^{(2)}=\frac{1}{gH_1}dt\wedge dx+\frac{r_5^2}{g}(\psi-\sin\psi\cos\psi)\sin\theta d\theta\wedge d\phi \tag{2.3}$$

Note that there is a 'Dirac string' singularity at $\psi=\pi$, the invisibility of which imposes quantization of Q_5. The isometry group of the background is

$$\begin{aligned} ISO(1,1)\times \mathrm{SO}(4) &\qquad a=1 \\ \mathrm{SO}(2,2)\times\mathrm{SO}(4) &\qquad a=0 \end{aligned}$$

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Writing the SO(4) as SU(2) $\times$ SU(2), it is the diagonal SU(2) that acts transitively on the two-spheres of constant ψ in (2.2). This subgroup will play an important role in what follows.

2.2 Spherical D3-branes

We will now discuss a class of spherically symmetric D3-brane probes, carrying worldvolume flux, in this background. We restrict our attention to branes whose worldvolume includes an S^2 embedded within S^3, parametrized by θ and ϕ in (2.2), whose size we allow to vary as a function of the other coordinates. In other words, the worldvolume geometry is an S^2 fibered over a 1+1 dimensional base. Such branes are 'spherically symmetric' in the sense that they preserve the SU(2) symmetry that acts on the S^2 fibre. Such configurations naturally generalize the known D3-brane solutions in the near-horizon region with $AdS_2 \times S^2$ geometry, where the size of the S^2 is constant [12 – 14]. The latter solutions will play a special role in what follows, as they will play the role of the attractor geometry with enhanced supersymmetry. We also allow general worldvolume gauge fields consistent with the SU(2) symmetry. This restricts the worldvolume gauge field to have an electric part $F_{\rm el}$ on the 1+1 dimensional base, a magnetic part $F_{\rm magn}$ with legs on the S^2 fiber, and no mixed components.

The terms contributing to the worldvolume action are then

$$S = -\mu_3 \int d^4\sigma e^{-\Phi} \sqrt{-\det(\hat{G}+F)} + \mu_3 \int F_{\rm el} \wedge \hat{C}^{(2)}_{\rm magn} + \mu_3 \int F_{\rm magn} \wedge \hat{C}^{(2)}_{\rm el} \tag{2.4}$$

where $\mu_3 = 1/((2\pi)^3 \alpha'^2)$ is the D3-brane charge density and a $\hat{}$ denotes a pullback to the worldvolume. Turning on $F_{\rm el}$ is necessary for stabilizing the contractible S^2 on which the brane is wrapped. With both $F_{\rm el}$ and $F_{\rm magn}$ turned on, the brane becomes a source for fundamental string charge (denoted by q) and D-string charge (denoted by p) as well. Let us first discuss the quantization conditions following from this. Requiring that the source terms for the electric NSNS and RR two-forms are properly quantized leads to[1]

$$q = \frac{\mu_3}{\mu_1} \int_{S^2} (\star \tilde{F}_{\rm el} + \hat{C}^{(2)}_{\rm magn}) \tag{2.5}$$

$$p = \frac{\mu_3}{\mu_1} \int_{S^2} F_{\rm magn}. \tag{2.6}$$

where $\star$ is the worldvolume Hodge star. We have defined a field $\tilde{F}$ as

$$\mu_3 \sqrt{-\det\, \hat{G}} \tilde{F}^{ab} = \frac{\delta S_{\rm BI}}{\delta F_{ab}}$$

The integrals are to be performed over the S^2 fibre. The equation of motion and Bianchi identity for the worldvolume gauge field imply that the charges are well-defined and independent of the position on the base. The fact that the fundamental string charge q receives

[1] We are ignoring here curvature corrections to this formula, not to mention further subtleties in defining charges in Ramond backgrounds. What we find is, however, consistent upon S-dualizing with the better understood quantization conditions in pure Neveu-Schwarz backgrounds, as we will see in section 6.

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a Wess-Zumino contribution from the second term in (2.5) has an important consequence in the near-horizon limit, where the S^3 becomes noncontractible, namely that q takes values in $\mathbf{Z}_{Q_5}$. The Wess-Zumino term is invariant under small gauge transformations of $C^{(2)}$, but shifts by a multiple of Q_5 under large gauge transformations. This is most easily seen by writing it as $\frac{\mu_3}{\mu_1}\int_B \hat{F}^{(3)}$ with B a 3-surface chosen such that $\delta B = S^2$. Different choices of B can differ by a map with nonzero winding number n around S^3 which, using the normalization of $F^{(3)}$ in (2.1), leads to an identification

$$q \sim q + nQ_5. \tag{2.7}$$

As we shall illustrate in more detail in section 6, this quantization condition is simply the S-dual version of the well-known D1-charge quantization in the background of $NS5$-branes. As long as the size of the S^2 fiber is sufficiently small, we expect our configurations to describe (p,q) strings expanded to form a 'fuzzy' D3-brane through a form of the Myers effect [25]. We will come back to this fuzzy sphere description in more detail shortly.

2.3 Action

We will now write out the action (2.4) in more detail, starting by fixing the worldvolume reparametrization invariance. It will be convenient to choose the worldvolume coordinates σ^a to coincide with (t,x,θ,ϕ). SU(2) invariance restricts the worldvolume scalars r,ψ and the electric field strength F_{tx} to be independent of θ,ϕ, while $F_{\theta\phi}$ should be proportional to $\sin\theta$. The quantization condition (2.6) leads to

$$F_{\theta\phi} = \frac{p\mu_1}{4\pi\mu_3}\sin\theta \tag{2.8}$$

where $\mu_1 = 1/(2\pi\alpha')$ is the D1-brane charge density. Substituting into the action and performing the θ,ϕ integrals leads to a consistent truncation of the original theory, resulting in an effective 1+1 dimensional action for a string-like object:

$$S = -\mu_1 \int dtdx \left[pe^{-\Phi}\sqrt{g\tilde{g}} - \frac{Q_5}{\pi}F_{tx}(\psi - \sin\psi\cos\psi) - \frac{p}{gH_1}\right] \tag{2.9}$$

where

$$\begin{aligned} g &\equiv (H_1H_5)^{-1}\left(1 - H^1H_5(\dot{r}^2 + r^2\dot{\psi}^2)\right)\left(1 + H_1H_5(r'^2 + r^2\psi'^2)\right) - F_{tx}^2 \\ \tilde{g} &\equiv 1 + \frac{r^4H_1H_5\sin^4\psi}{p^2\pi^2\alpha'^2}. \end{aligned}$$

Here, we denoted the time derivative by a $\dot{}$ and the x derivative by a $'$.

We expect such configurations to represent (p,q) strings expanded to form a 'fuzzy' D3-brane through a form of the Myers effect [25]. In an alternative description, they should arise as noncommutative fuzzy sphere solutions in the worldvolume theory of p coinciding D-strings, with the $\Upsilon(1)$ part of the worldvolume field strength turned to induce the fundamental string charge q. From the analysis of [25] one expects the latter description (as a perturbative expansion in the matrix-valued coordinates) to be valid as long as the S^2 radius in string units is smaller than p, or

$$\frac{\sqrt{H_1 H_5} r^2 \sin^2\psi}{\pi\alpha'} \ll p. \tag{2.10}$$

In this limit, we can expand $\sqrt{\tilde g}$ in (2.9):

$$\sqrt{\tilde g} = 1 + \frac{r^4 H_1 H_5 \sin^4\psi}{2p^2\pi^2\alpha'^2} + \dots$$

In appendix A we show that the noncommutative worldvolume theory of p coinciding D-strings allows a fuzzy sphere solution and, expanding around it, we obtain precisely the action (2.9) in the limit (2.10).

2.4 Hamiltonian

The canonical momenta $P_r \equiv \frac{\partial\mathcal{L}}{\partial\dot r}, P_\psi \equiv \frac{\partial\mathcal{L}}{\partial\dot\psi}$ and $\Pi \equiv \frac{\partial\mathcal{L}}{\partial F_{tx}}$ following from the action (2.9) are given by:

$$\begin{aligned}
P_r &= \mu_1 p e^{-\Phi}\sqrt{\frac{\tilde g}{g}}\left(1 + H_1 H_5 (r'^2 + r^2\psi'^2)\right)\dot r \\
P_\psi &= \mu_1 p e^{-\Phi}\sqrt{\frac{\tilde g}{g}}\left(1 + H_1 H_5 (r'^2 + r^2\psi'^2)\right) r^2\dot\psi \\
\Pi &= \mu_1\left(p e^{-\Phi}\sqrt{\frac{\tilde g}{g}} F_{tx} + \frac{Q_5}{\pi}(\psi - \sin\psi\cos\psi)\right)
\end{aligned} \tag{2.11}$$

We can define the phase-space lagrangian density $\mathcal{L}$ as

$$\mathcal{L} = \dot x P_x + \dot r P_r + \dot\psi P_\psi + \dot A_\rho \Pi - L = \left(\dot x P_x + \dot r P_r + \dot\psi P_\psi + F_{t\rho}\Pi - L\right) - A_t \Pi' \tag{2.12}$$

where we have done a partial integration in the second equality. The quantity in brackets can be identified as the (improved) Hamiltonian density, while the second term imposes the Gauss law constraint $\Pi' = 0$ for the worldvolume gauge field. The quantization condition (2.5) on the fundamental string charge fixes the the integration constant in this equation in terms of q:

$$\Pi = q\mu_1.$$

Substituting in (2.12) and restricting attention to static configurations

$$P_r = P_\psi = 0$$

one finds for the Hamiltonian

$$H = \mu_1 \int dx \left[\frac{Q_5}{\pi}\sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}\sqrt{\frac{1}{H_1 H_5} + r'^2 + r^2\psi'^2} - \frac{p}{g H_1}\right]. \tag{2.13}$$

Here, we have defined functions $\Delta_1, \Delta_2, \Delta_3$ that will reappear frequently in what follows:

$$\begin{aligned}
\Delta_1 &\equiv \sin\psi\cos\psi - (\psi - \frac{q}{Q_5}\pi) \\
\Delta_2 &\equiv \frac{r^2}{r_5^2} H_5 \sin^2\psi \\
\Delta_3 &\equiv p\frac{\pi}{gQ_5}\sqrt{\frac{H_5}{H_1}}.
\end{aligned} \tag{2.14}$$

We also record, for later use, the expression for the worldvolume electric field

$$F_{tx} = \sqrt{\frac{1}{H_1 H_5} + r'^2 + r^2\psi'^2}\,\frac{\Delta_1}{\sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}}. \tag{2.15}$$

The total energy (2.13) is the sum of two competing contributions: the first term represents the energy of a static string with variable tension

$$T(r,\psi) = \mu_1 \frac{Q_5}{\pi}\sqrt{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}. \tag{2.16}$$

The second term represents the Coulomb energy of p D-strings placed in the electric RR potential $C^{(2)}_{\rm el}$ produced by the D-strings in the background. Both terms cancel precisely for pure D1-string probes ($q = 0,\ \psi = 0$) placed parallel to the D1-strings in the background (i.e. at constant r). This solution can be seen as the zero-energy ground state of the effective string description. Turning on the fundamental string charge q amounts to turning on a central charge in the worldvolume superalgebra, as we shall presently see, and leads to a class of BPS-solutions that can be seen as worldvolume solitons. When the D1-charge p is zero, the energy doesn't contain the Coulomb contribution and a BPS bound on the energy can be derived using standard methods [24]. We will comment on this special case later on. When $p \neq 0$, the energy is not manifestly a sum of positive contributions, but we will see that it is possible to rewrite it in an equivalent form suitable for deriving a BPS-type bound.

2.5 BPS equations and gradient flow

The Hamiltonian leads to second order equations for $r(x), \psi(x)$, which will reduce to a first order system for BPS configurations. Some intuition can be gained by viewing the Hamiltonian as an action functional describing geodesic motion of an effective particle with a position dependent mass $m(r,\psi)$ in a curved three dimensional space with metric $dl^2 = \frac{1}{H_1}dx^2 + H_5(dr^2 + r^2 d\psi^2)$ in the presence of a gauge potential $A = -\frac{p}{gH_1}dx$:

$$H = \frac{Q_5}{\pi}\mu_1 \int d\rho \left[m(r,\psi)\sqrt{\frac{\dot{x}^2}{H_1} + H_5(\dot{r}^2 + r^2\dot{\psi}^2)} - \frac{p\pi}{gQ_5 H_1}\dot{x} \right] \tag{2.17}$$

with

$$m(r,\psi) = \sqrt{\frac{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}{H_5}}$$

We have introduced an arbitrary parameter ρ on the worldline of the effective particle, and denoted the ρ-derivative by a $\dot{}$, hopefully without causing confusion with the time derivative (all configurations considered henceforth will be static). We recover the earlier expression (2.13) after choosing $\rho = x$. We can write a classically equivalent system without the square root by introducing an auxiliary einbein $e(\rho)$ on the worldline:

$$H = \frac{Q_5}{\pi}\mu_1 \int d\rho \left[\frac{1}{2e}\left(\frac{x'^2}{H_1} + H_5(r'^2 + r^2\psi'^2) \right) + \frac{e}{2}\frac{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}{H_5} - \frac{p\pi}{gQ_5 H_1}x' \right]$$

Solving for for the einbein leads back to (2.17). This expression can, up to a boundary term, be written as a sum as a sum of squares. This can be seen by using the definitions (2.14) and observing that

$$\Delta_1^2 + \Delta_2^2 = (\partial_r(rZ))^2 + (\partial_\psi Z)^2$$

where we have defined a function Z as

$$Z \equiv \sin\psi - (\psi - \frac{q}{Q_5}\pi)\cos\psi + \frac{a}{3}\left(\frac{r}{r_5}\right)^2 \sin^3\psi \tag{2.18}$$

The Hamiltonian can then be written as

$$\begin{aligned} H = \frac{Q_5}{\pi}\mu_1 \int d\rho \Bigg[&\frac{1}{2}\left(\sqrt{\frac{H_5}{e}}\dot{r} \pm \sqrt{\frac{e}{H_5}}\partial_r(rZ) \right)^2 + \frac{1}{2}\left(\sqrt{\frac{H_5}{e}}r\dot{\psi} \pm \sqrt{\frac{e}{H_5}}\partial_\psi Z) \right)^2 \\ &+ \frac{1}{2eH_1}\left(\dot{x} - \frac{p\pi}{gQ_5}e \right)^2 \mp \frac{d}{d\rho}(rZ) \Bigg] \end{aligned} \tag{2.19}$$

The energy is bounded below by a total derivative, and configurations saturating the bound will automatically obey the equations of motion. This gives the desired system of first order BPS equations. The quantity $\frac{Q_5}{\pi}\mu_1 rZ$ plays the role of a central charge in the worldvolume superalgebra [26, 27, 22].

Let us first consider the case $p \neq 0$. It will be convenient to define dimensionless parameters $\tilde{x}$, $\tilde{r}$ as

$$\tilde{x} \equiv \frac{gQ_5}{p\pi r_5}x; \qquad \tilde{r} \equiv \frac{r}{r_5}.$$

Choosing $\rho = \tilde{x}$ in (2.19), the BPS equations reduce to $e = r_5$ and

$$\begin{aligned} \dot{\tilde{r}} &= -\frac{1}{H_5}\partial_{\tilde{r}}(\tilde{r}Z) \\ \dot{\psi} &= -\frac{1}{H_5\tilde{r}^2}\partial_\psi(\tilde{r}Z). \end{aligned} \tag{2.20}$$

Here we have chosen the upper sign in (2.19) without loss of generality, as the equations with the other sign choice are related by a reflection $\tilde{x} \to -\tilde{x}$. The equations describe a gradient flow with potential function $\tilde{r}Z$ on a space with metric $H_5(d\tilde{r}^2 + \tilde{r}^2 d\psi^2)$. It will be useful at times to change the independent variable to $\tilde{r}$ and obtain equations for $\tilde{x}(\tilde{r}), \psi(\tilde{r})$:

$$\begin{aligned} \frac{d\tilde{x}}{dr} &= -\frac{H_5}{\partial_{\tilde{r}}(\tilde{r}Z)} \\ \tilde{r}\frac{d\psi}{d\tilde{r}} &= \frac{\partial_\psi Z}{\partial_{\tilde{r}}(\tilde{r}Z)}. \end{aligned} \tag{2.21}$$

The total energy of the solutions is

$$E = \frac{Q_5 r_5}{\pi} \mu_1 \left[\tilde{r} Z\right]_{\tilde{x}_f}^{\tilde{x}_i}. \tag{2.22}$$

We now turn to the case $p = 0$. In this case, we cannot choose ρ to be proportional to x anymore. Instead, we can take $\rho = \tilde{r}$, and the BPS equations become

$$\begin{aligned} \frac{dx}{dr} &= 0 \\ \tilde{r}\frac{d\psi}{d\tilde{r}} &= \frac{\partial_\psi Z}{\partial_{\tilde{r}}(\tilde{r} Z)}. \end{aligned} \tag{2.23}$$

Comparing with (2.21), we see that the flows in the $p = 0$ case are simply the flows for any $p \neq 0$ projected onto a surface of constant x.

2.6 Worldvolume geometry

We collect here for later convenience also the formulae for the worldvolume fields for BPS solutions satisfying (2.20). Making use of the relation $H_1 H_5 (r'^2 + r^2 \psi'^2) = \frac{\Delta_1^2 + \Delta_2^2}{\Delta_3^2}$ satisfied by these solutions, the induced metric and the gauge field on the worldvolume can be written as

$$\begin{aligned} d\hat{s}^2 &= (H_1 H_5)^{-1/2} \left(-dt^2 + \frac{\Delta_1^2 + \Delta_2^2 + \Delta_3^2}{\Delta_3^2} dx^2 \right) + (H_1 H_5)^{1/2} r^2 \sin^2\psi \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \\ F &= (H_1 H_5)^{-1/2} \frac{\Delta_1}{\Delta_3} dt \wedge dx + (H_1 H_5)^{1/2} r^2 \sin^2\psi \frac{\Delta_3}{\Delta_2} \sin\theta d\theta \wedge d\phi \end{aligned} \tag{2.24}$$

One easily derives the components of the open string metric $g^o_{\mu\nu} = g_{\mu\nu} - \mathcal{F}_{\mu\rho} g^{\rho\sigma} \mathcal{F}_{\sigma\nu}$:

$$d\hat{s}_o^2 = \frac{\Delta_2^2 + \Delta_3^2}{\sqrt{H_1 H_5}} \left(-\frac{dt^2}{\Delta_1^2 + \Delta_2^2 + \Delta_3^2} + \frac{dx^2}{\Delta_3^2} \right) + \frac{\sqrt{H_1 H_5}(\Delta_2^2 + \Delta_3^2) r^2 \sin^2\psi}{\Delta_2^2} \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \tag{2.25}$$

3. The near-horizon limit and attractor flows

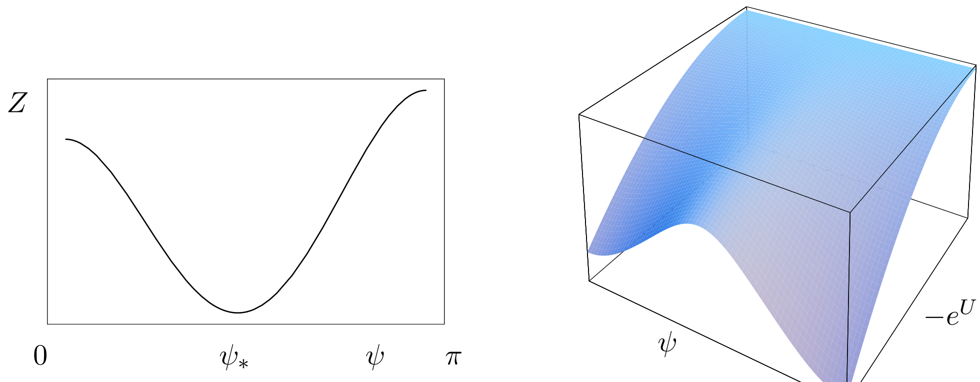
We shall now discuss the solutions of the BPS equations (2.20) in the near-horizon limit of the background geometry obtained by putting $a = 0$ in the equations above. It is in this limit that the BPS flow equations most closely resemble the attractor flows for supersymmetric black holes in $N = 2$ supergravity.

3.1 Flow equations

A first observation is that, in the near-horizon limit, the function Z depends on ψ alone:

$$Z = Z(\psi) = \sin\psi - \left(\psi - \frac{q}{Q_5}\pi \right) \cos\psi.$$

As we saw in (2.7), q is a $\mathbf{Z}_{Q_5}$ valued charge and we will take $0 \leq q < Q_5$ in what follows. The function Z is then a positive function with a single minimum at $\psi = \frac{q}{Q_5}\pi \equiv \psi_*$ and



(a) (b)

Figure 1: (a) The function Z. (b) The potential V.

maxima at the boundary points $\psi = 0, \pi$ (see figure 1(a)). It will be convenient to rewrite the equations derived in the previous section in terms of a coordinate U defined as

$$\tilde{r} \equiv e^U .$$

Assuming $p \neq 0$, it is a consistent truncation to substitute the value $e = r_5$, which solves the equation of motion for $\tilde{x}$, into (2.19) as long as we impose the equation for e as a constraint. Choosing again $\rho = \tilde{x}$, this truncated energy function takes the form:

$$\begin{aligned} H &= \frac{\mu_1 Q_5 r_5}{2\pi} \int d\tilde{x} \left[\dot{U}^2 + \dot{\psi}^2 + e^{2U}(Z^2 + \partial_\psi Z^2)\right] \\ &= \frac{\mu_1 Q_5 r_5}{2\pi} \int d\tilde{x} \left[\left(\dot{U} \pm \partial_U (e^U Z)\right)^2 + \left(\dot{\psi} \pm \partial_\psi (e^U Z)\right)^2\right] \pm \frac{\mu_1 Q_5 r_5}{\pi} [e^U Z]^{\tilde{x}_i}_{\tilde{x}_f} \end{aligned} \tag{3.1}$$

It describes a particle moving on the (U, ψ) strip with flat metric in an inverted potential $V = -e^{2U}(Z^2 + \partial_\psi Z^2)$ (see figure 1(b)). The constraint from the equation for e becomes

$$\dot{U}^2 + \dot{\psi}^2 - e^{2U}(Z^2 + \partial_\psi Z^2) = 0. \tag{3.2}$$

It states that the conserved total 'energy' of the effective particle is zero and can be imposed as an initial condition. Choosing again the upper sign, the BPS equations are

$$\dot{U} = -\partial_U (e^U Z) \tag{3.3}$$

$$\dot{\psi} = -\partial_\psi (e^U Z). \tag{3.4}$$

Note that solutions to these equations obey the constraint (3.2). The energy is given by

$$E = \frac{Q_5 r_5}{\pi} \mu_1 \left[e^U Z\right]^{\tilde{x}_i}_{\tilde{x}_f} . \tag{3.5}$$

3.2 Solutions

The system (3.3), (3.4) describes a gradient flow on the flat (U, ψ) strip with potential function $e^U Z$. The flow is directed towards the minimum of Z, since the second equation implies that

$$\dot{Z} = -e^U (\partial_\psi Z)^2 \leq 0$$

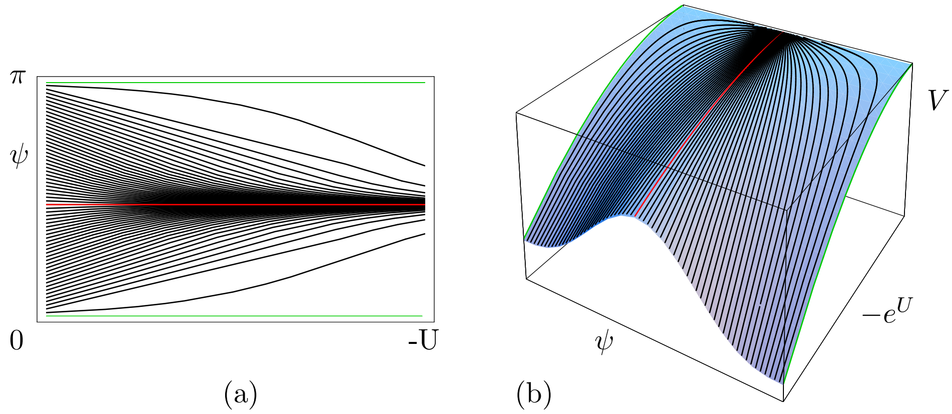


Figure 2: (a) Gradient flows in the $(-U, \psi)$ plane. The red line is the attractive fixed point, the green lines are repulsive fixed points. (b) BPS trajectories 'shoot for the top' of the inverted potential.

Hence there is an attractive fixed point at

$$\psi_* = \frac{q}{Q_5}\pi$$

where Z is minimal and takes the value $Z_* = \sin \frac{q}{Q_5}\pi$. The maxima of Z at $\psi = 0$ or π represent repulsive fixed points. Furthermore, U is a decreasing function since $\dot{U} = -e^U Z \leq 0$ and the flows will eventually end up at $U_* = -\infty$, corresponding to the 'horizon' $r = 0$ in Poincaré coordinates. The BPS solutions correspond to particle trajectories where the initial conditions are tuned such that the particle reaches the top of the inverted potential asymptotically. Figure 2 illustrates these aspects of the gradient flow.

In terms of the embedding of the fuzzy (p, q) string, these equations tell us that, if we fix one endpoint of the string somewhere in AdS_3 and specify some value of the fuzzy radius at this point, while letting the other end 'flap in the breeze', the string will eventually reach $r = 0$ at $x = \infty$, and the S^2 radius will approach the value $\sqrt{r_1 r_5} Z_*$. The equations (3.3), (3.4) can also be solved exactly and we will now discuss the different types of solutions in more detail. We are interested in complete flows where the string starts out at the boundary of of AdS_3 and not somewhere in the interior, which would be forbidden by charge conservation. We will use translation invariance in the x-direction to make the starting point at $r = \infty$ correspond to $x = 0$, which fixes the integration constant in (3.3). We also note that the equations (3.3), (3.4) are invariant under changing $\psi \to \pi - \psi$ and $q \to Q_5 - q$.

3.2.1 Attractive fixed point: $AdS_2 \times S^2$ branes

Of special importance is the solution where ψ takes on the constant attractor value ψ_* everywhere:

$$\begin{aligned} \psi &= \psi_* \\ \tilde{r} &= \frac{1}{\sin \psi_* \tilde{x}} \end{aligned} \tag{3.6}$$

Generic solutions of (3.3), (3.4) approach this one for large $\tilde{x}$ and we will call this the 'attractor solution'. The tension (2.16) for this solution is constant and given by

$$T^{\rm attr}_{(p,q)} = \mu_1 \sqrt{p^2 e^{-2\phi} + \left(\frac{Q_5}{\pi}\sin\frac{q\pi}{Q_5}\right)^2}$$

The induced metric (2.24) for this solution is $AdS_2 \times S^2$, and we will see in section 5 (see also [14]) that it is 1/2-BPS, preserving 4 out of 8 Poincaré supercharges and 4 out of 8 conformal supercharges of the near-horizon background. In the induced metric on the worldvolume (2.24), the radii of the S^2 and AdS_2 factors are different and given by

$$R_{\rm AdS_2} = L\frac{T^{\rm attr}_{(p,q)}}{T^{\rm attr}_{(0,q)}}; \qquad R_{\rm S^2} = LZ_* \tag{3.7}$$

with $L \equiv \sqrt{r_1 r_5}$. An interesting feature of the attractor solution is that, in the open string metric (2.25), the S^2 and AdS_2 radii become equal and are given by

$$R^o_{\rm AdS_2} = R^o_{\rm S_2} = L\frac{\sqrt{T^{\rm attr\ 2}_{(p,q)}\sin^2\psi_* + T^{\rm attr\ 2}_{(p,0)}\cos^2\psi_*}}{T^{\rm attr}_{(0,q)}}. \tag{3.8}$$

This property is consistent with an argument made in the S-dual system in [15].

3.2.2 Repulsive fixed point: AdS_2 branes

Another special solution to the equations (3.3), (3.4) which deserves to be mentioned is obtained by taking ψ to be constant and equal to 0 or π, the maxima of Z. These solutions correspond to the repulsive fixed points of the flow equations, and small supersymmetric deformations cause the flow to move away from them and end up at the attractive fixed point. One could also wonder whether these solutions are physical, since the S^2 fiber has collapsed to a point and it is not clear whether the D3-brane description is still reliable. Nevertheless, in the description as a noncommutative theory on coinciding D-strings, they simply correspond to the solution with commuting matrices discussed in appendix A, and that formulation should be reliable. In section 4, we shall show that these solutions arise as 'spikes' on a D3-brane in the limit that the D3-brane is moved away to infinity. These observations suggest that we should not discard these solutions. The $\psi = 0$ solution has the tension (2.16) of a (p, q) string in flat space

$$T_{(p,q)} = \mu_1\sqrt{p^2 e^{-2\phi} + q^2}$$

(for $\psi = \pi$ we have to replace q by $Q_5 - q$). The radial coordinate is given by

$$\tilde{r} = \frac{1}{\psi_*}\frac{1}{\tilde{x}} \tag{3.9}$$

and the resulting worldvolume geometry is AdS_2, with the radius in the induced and open string metrics given by

$$R_{\rm AdS_2} = L\frac{T_{(p,q)}}{T_{(0,q)}} \qquad R^o_{\rm AdS_2} = L\frac{T_{(p,0)}}{T_{(0,q)}}$$

These solutions can be shown to preserve half of the near-horizon supersymmetries as well [14].

These solutions represent a (p, q) string which has not expanded to form a D3-brane. The situation is reminiscent of the case of giant gravitons [28, 29], where there are also different supersymmetric solutions representing expanded and non-expanded configurations carrying the same charges. This is perhaps not so surprising, since the coupling which allows the two-sphere to be stabilized in our case (i.e. the coupling of the electric worldvolume field to the RR background) is T-dual to the coupling allowing giant gravitons to expand (i.e. the coupling of an angular momentum to the RR background).

3.2.3 General flows

The general solution to the flow equations (3.3), (3.4) is

$$\begin{aligned} \psi &= \psi_* + \frac{1}{C_1 \tilde{x} + C_2} \\ \tilde{r} &= C_1 \frac{\psi - \psi_*}{\sin\psi} \end{aligned} \tag{3.10}$$

The solution breaks scale and translation symmetry of the background, and solutions with different values of the integration constants C_1, C_2 are related by the action of these broken generators. The initial condition that $r = \infty$ at $\tilde{x} = 0$ fixes the constant C_2 in (3.10). We see from (3.10) that the flows must start at either $\psi = 0$ or $\psi = \pi$. Due to the above mentioned symmetry $\psi \to \pi - \psi$, $q \to Q_5 - q$ we can restrict attention to flows starting at $\psi = 0$. These have $C_2 = -1/\psi_*$, C_1 negative and $\psi \leq \psi_*$ everywhere. The constant C_1 represents the value of $r \sin\psi$ at the boundary of AdS_3 and can be interpreted as an asymptotic modulus. As we will see in section 5, the generic solution is 1/4 BPS, preserving 4 out of the 16 real supercharges of the near-horizon region. It preserves half of the Poincaré supersymmetries but breaks all of the conformal ones. A plot of various flows in coordinate space is shown in figures 3(a) and 5(a).

The attractor solution (3.6) and the repulsive solution (3.9) are obtained in the limits $C_1 \to -\infty$, $C_1 \to 0$ respectively, where scale invariance is restored. Hence we see that the general 1/4 BPS flows represent worldvolume solitons that interpolate between the 1/2 BPS repulsive solution at $\tilde{x} = 0$ and the 1/2 BPS attractor solution at $\tilde{x} = \infty$. The tension (2.16) also interpolates between $T_{(p,q)}$ and $T^{\mathrm{attr}}_{(p,q)}$ as shown in figure 4.

The energy of the solutions can be read off from (3.5). Since $e^U Z$ becomes zero for $\tilde{x} \to \infty$ it is given by

$$E = \frac{\mu_1 Q_5 r_5}{\pi} [e^U Z]_{|\tilde{x}=0}$$

The energy is divergent due to the fact that the string stretches all the way to the boundary of AdS_3. The variable ψ approaches zero near the boundary and introducing a cutoff at a small value of ψ we find

$$\begin{aligned} E &= \frac{\mu_1 Q_5}{\pi} \lim_{\psi \to 0} r(\psi) \left(\sin\psi - (\psi - \psi_*) \cos\psi\right) \\ &= q \mu_1 \lim_{\psi \to 0} r(\psi) \end{aligned} \tag{3.11}$$

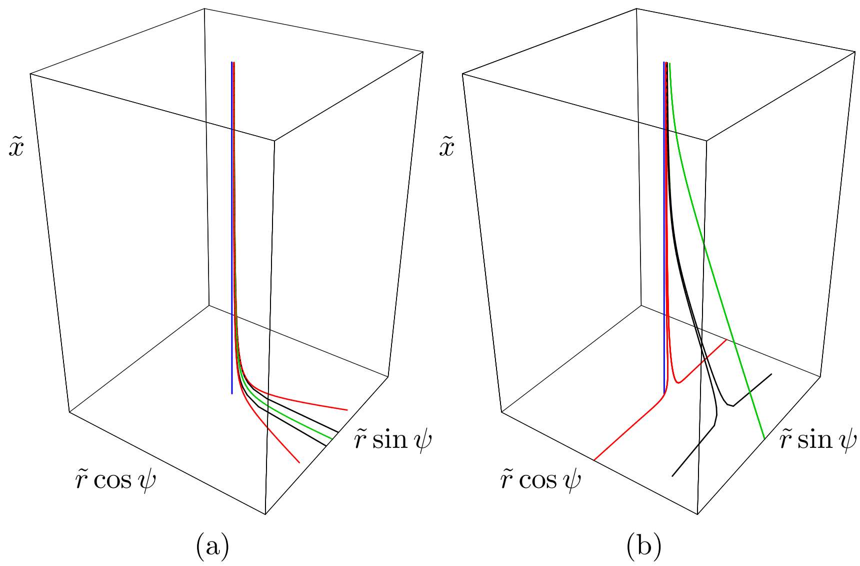


Figure 3: Flow plots in $(\tilde{r}\cos\psi, \tilde{r}\sin\psi, \tilde{x})$ coordinates. Each brane configuration forms a 'tube' whose cross-section is an S^2, represented by two points on opposite sides of the $\tilde{r}\sin\psi$ axis. The vertical blue line represents the D1-D5 string in the background. (a) Solutions in the near-horizon geometry: the black curve denotes a generic flow, while the green and red curves represent the repulsive and attractive fixed point solutions with $\psi = 0$ and $\psi = \psi_*$ respectively. (b) Solutions in the full geometry: the generic solution represents a (p, q) string 'spike' ending on a D3-brane transverse to the D1-D5 string in the background. The attractive and repulsive fixed point solutions arise from the limits where the transverse distance of the D3-brane is taken to zero or infinity respectively.

We see that the energy is equal to the energy of q fundamental strings stretched along the radial direction, perpendicular to the D1-D5 string in the background. We note in particular that the regularized energy doesn't depend on the asymptotic modulus C_1 and the solutions are degenerate in this sense. The D-string charge p doesn't enter into the expression for the energy because a D-string probe is mutually BPS with the branes in the background.

Let us also comment on the solutions with zero D1-charge $p = 0$, which, as we saw in (2.23), are obtained by projecting the $p \neq 0$ solutions onto a surface of constant x, as illustrated in figure 5(a). The BPS equation (2.23) is the same one that arises in the description of D3-branes with electric field in a D5-brane background and has been studied in the literature before [24]. It is a special case of a class of generalized 'baryon vertex' solutions that were studied in [20 – 23]. It was observed in these works that such solutions approach a special solution where the angle ψ is constant. We can now reinterpret this property as a special (albeit less transparent) case of the attractor mechanism. It would be interesting to study the question of supersymmetry enhancement at the attractor point in these examples.

Our discussion of the solutions has been entirely in the Poincaré patch, and it would be interesting to study how the geometry extends into global AdS_3 in more detail. While both the attractive and repulsive fixed point solutions represent static configurations with

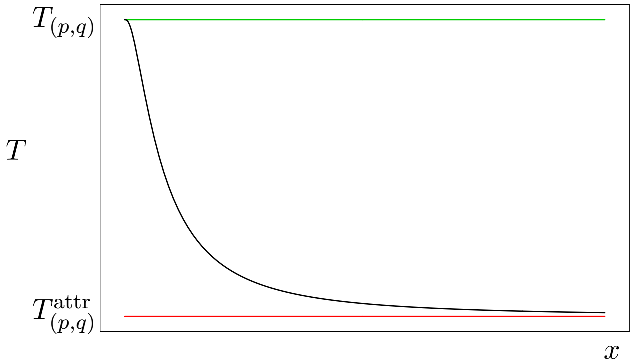


Figure 4: The tension of a generic flow solution (black line) interpolates between the tension of the repulsive fixed point (green line) and attractive fixed point (red line) solutions.

respect to global time, this no longer the case for the general solution which will interpolate between the two in a time dependent manner.

3.3 'Entropy' function

For attractor black holes, the attractor geometry where the moduli take their constant fixed-point values can also be derived by extremizing an entropy function [11], which, at the minimum, coincides with the physical entropy of the black hole. In our open string example, a similar role is played by the energy function (2.19) evaluated for an ansatz where the worldvolume geometry is $AdS_2 \times S^2$. Extremizing this function yields the value of the AdS_2 and S^2 radii in terms of the charges, as we shall presently illustrate. It's not clear whether it can be related to an entropy contained in the open string degrees of freedom, which is one of the main questions raised by our example.

We start by defining new target space coordinates V_1, V_2:

$$\begin{aligned} V_1 &= \sqrt{1+(ux)^2} \\ V_2 &= \sin\psi \end{aligned}$$

with $u \equiv \frac{r}{L^2}$, $L \equiv \sqrt{r_1 r_5}$. If (V_1, V_2) take on constant values (v_1, v_2), the induced worldvolume metric is $AdS_2 \times S^2$:

$$d\hat{s}^2 = L^2\left[-u^2dt^2 + \frac{v_1^2}{u^2}du^2 + v_2^2(d\theta^2 + \sin^2\theta d\phi^2)\right].$$

Hence v_1 and v_2 are the radii of AdS_2 and S^2 in units of L. In analogy with the entropy function for black holes, we define the entropy function $F(v_1, v_2; p, q)$ to be the Hamiltonian density (2.19) evaluated at constant values of the scalar fields V_1, V_2:

$$H \equiv \int du F(v_1, v_2; p, q).$$

One finds

$$F(v_1, v_2; p, q) = L^2\left[T(v_2)v_1 - T^{\rm attr}_{(p,0)}\sqrt{v_1^2 - 1}\right]$$

where T is the tension given in (2.16). Extremizing the entropy function gives the correct values for v_1, v_2 at the attractor point. The variation with respect to v_2 tells us to extremize the tension and determines

$$v_2 = \sin\psi_*.$$

Variation with respect to v_1 yields

$$v_1 = \frac{T^{\rm attr}_{(p,q)}}{T^{\rm attr}_{(0,q)}}$$

in agreement with the earlier result (3.7). We note that the value of the entropy function at the minimum is independent of the probe D1-charge and is given by

$$F = L^2 T^{\rm attr}_{(0,q)}.$$

In the case of attractor black holes, the entropy function formalism greatly facilitates finding the attractor solution in the presence of higher derivative corrections [11], and it would be interesting to see if the same is true here.

3.4 Comparison with the attractor mechanism for black holes

The attractor mechanism described above is remarkably similar to the familiar attractor mechanism governing supersymmetric black holes in $N = 2$ supergravity theories with vector multiplets [1–3]. Let us pause for a moment to identify similar quantities appearing in both systems.

Spherically symmetric attractor black holes in $N = 2$ supergravity theories are described by an effective particle action and constraint of the form (3.1), (3.2) [30, 31]. The flow equations that describe the evolution of the spacetime metric and the moduli take the form of a gradient flow analogous to (3.3), (3.4):

$$\begin{aligned} \dot{U} &= -\partial_U(e^U|Z|) \\ \dot{z^a} &= -g^{a\bar{b}}\partial_{\bar{b}}(e^U|Z|). \end{aligned} \tag{3.12}$$

Here, the function U appears in the metric ansatz $ds^2 = -e^{2U}dt^2 + e^{-2U}d\mathbf{x}^2$, the z^a are complex vector multiplet moduli and $g_{a\bar{b}}$ is the moduli space metric. The flow parameter is proportional to the inverse of the radial coordinate. The function Z is the graviphoton charge, which plays the role of the central charge in the $N = 2$ superalgebra. The equations describe a flow towards a minimum of Z, which becomes proportional to the black hole horizon radius. General solutions interpolate between the Minkowski vacuum at asymptotic infinity and an attractor solution where the moduli take on constant values and the geometry is the $AdS_2 \times S^2$ Bertotti-Robinson solution near the horizon. The general solution preserves 4 supersymmetries and interpolates between maximally supersymmetric vacua that preserve all 8 supersymmetries. The attractor geometry can also be derived from extremizing an 'entropy function' whose value at the extremum is the black hole entropy.

It's easy to draw parallels with our open string example. The variable U is now related to the time component of the induced worldvolume metric: $\hat{g}_{tt} \sim -e^{2U}$. The vector multiplet moduli are replaced in our example by a single real field ψ, which is related to the size of the S^2 fiber. The role of the graviphoton charge is played by the real, positive function Z, which, at its minimum, is proportional to the size of the S^2.[2] General flows preserve 4 supersymmetries and interpolate between solutions where the supersymmetry is enhanced to 8 supercharges: at infinity, an AdS_2 geometry where the S^2 has collapsed, and near $r = 0$, an $AdS_2 \times S^2$ geometry. The latter 'attractor solution' can also be obtained by extremizing an 'entropy' function as we saw in the previous paragraph.

4. Solutions in the asymptotically flat background

We now describe how the above solutions extend to the full, asymptotically flat D1-D5 geometry. This will help clarify the interpretation of our brane solutions as intersecting D3-brane 'spike' configurations embedded in the D1-D5 background. It will also provide a physical interpretation of the asymptotic modulus C_1 encountered in the near-horizon solution (3.10).

The BPS equations are now given by (2.20) for $a = 1$ and can still be solved analytically. For this, it's convenient to switch to ψ as the independent variable and solve for $\tilde{r}(\psi), \tilde{x}(\psi)$. The general solution satisfies

$$\begin{aligned} \tilde{r} \sin\psi &= C_1(\psi - \psi_* - a\tilde{r}^2 \sin\psi \cos\psi) \\ \tilde{x} &= \frac{1}{\tilde{r}\sin\psi} - \frac{C_2}{C_1} \end{aligned} \tag{4.1}$$

with $C_1,\ C_2$ integration constants that reduce to the previously introduced ones (3.10) in the $a \to 0$ limit. The first equation is a quadratic equation for $\tilde{r}$ and the solutions consist of two branches:

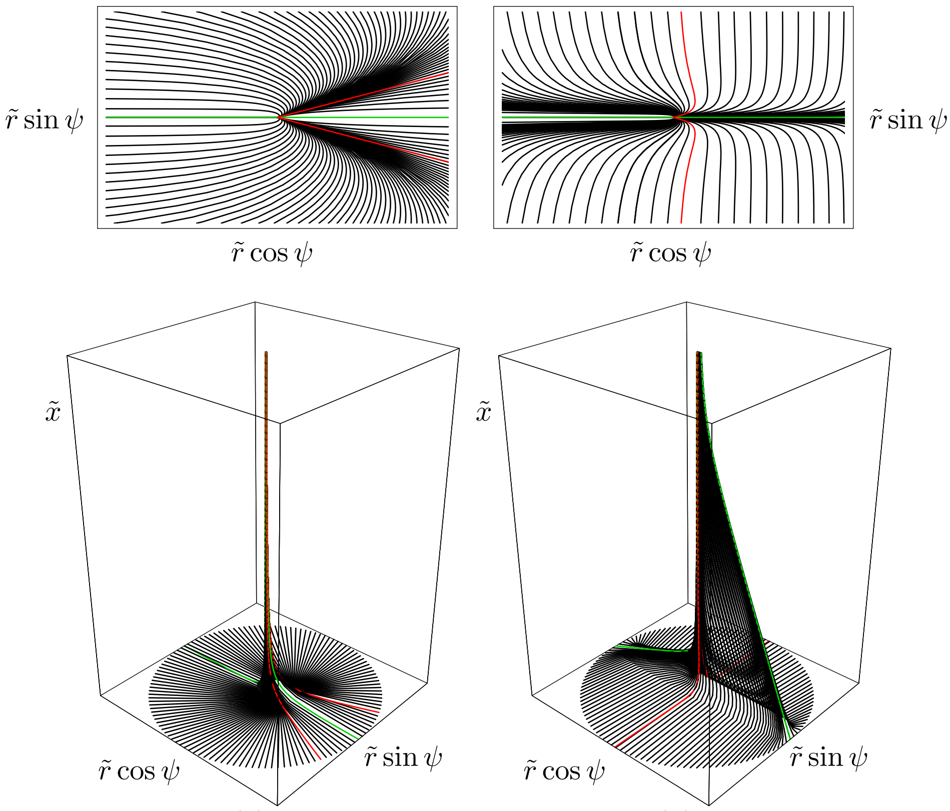
$$\tilde{r}_\pm = -\frac{1}{2aC_1 \cos\psi}\left[1 \pm \sqrt{1 + 4aC_1^2(\psi - \psi_*)\cot\psi}\right]$$

The two branches join at the point where the argument of the square root becomes zero. In the near horizon limit $a \to 0$, only the $-$ branch survives. The full solution describes a curve starting out at $\psi = \pi/2,\ r = \infty$ in the asymptotically flat region and approaching the solutions of the previous section near $r = 0$. One should note that, near $r = \infty$, the radius of the S^2 grows like r^2 and the solution approaches a flat D3-brane transverse to the D1-D5 string in the background. The transverse distance ΔY between the D3-brane and the D1-D5 string is given by the limiting value of $r\cos\psi$ as ψ approaches $\pi/2$ and is related to the integration constant C_1, which played the role of an asymptotic modulus in the near-horizon region:

$$\Delta Y = \lim_{\psi \to \pi/2} |r \cos\psi| = 1/|C_1|.$$

The general solution can be interpreted as a (p, q) string running between this D3-brane and the D1-D5 string in the background. This is illustrated in figures 3(b) and 5(b). The

[2] One should note however that the function Z is not quite the same as the worldvolume central charge, which is instead given by $e^U Z$.



(a) (b)

Figure 5: A sampling of flow solutions plotted in coordinate space (a) in the near-horizon region and (b) in the full geometry. The upper figure shows the flows projected to the $(\tilde{r}\cos\psi, \tilde{r}\sin\psi)$ plane (as appropriate for the solutions with vanishing D1-charge $p = 0$). The red curve represents the attractor solution, the green curves are the repulsive fixed point solutions with $\psi = 0$ and $\psi = \pi$.

energy (2.22) of the solutions contains a divergent term as well as a finite one:

$$E = \frac{4\pi\mu_3}{g} \lim_{\psi\to\pi/2} \left[\frac{(r(\psi)\sin\psi)^3}{3} + r_5^2(r(\psi)\sin\psi)\right] + (\frac{Q_5}{2} - q)\mu_1\Delta Y.$$

The divergent term is the energy of a flat D3-brane transverse to the D1-D5 string cut off at a large radius $r_f = r\sin\psi$: $\mu_3 \int e^{-\Phi}\sqrt{-g} = \frac{4\pi\mu_3}{g}\int_0^{r_f} drr^2(1 + (r_5/r)^2)$. The finite term represents the energy of $\frac{Q_5}{2} - q$ fundamental strings stretched over a distance ΔY perpendicular to the D1-D5-string.

We can also identify the solutions that reduce to the attractive and repulsive fixed points in the near-horizon region. These are obtained by taking $C_1 \to \infty$ and $C_1 \to 0$ respectively and correspond to putting the D3-brane at $r = 0$ or $r \to \infty$. Let's start with the latter case, where the D3-brane has moved off to infinity, leaving behind a (p, q) string with the S^2 shrunk to zero size. The explicit solution reads

$$\psi = 0 \quad x = \frac{r_5^2 p}{gq}\frac{1}{r} - a\frac{p}{gq}r + C \quad F_{tx} = \frac{gq}{pH_5} \tag{4.2}$$

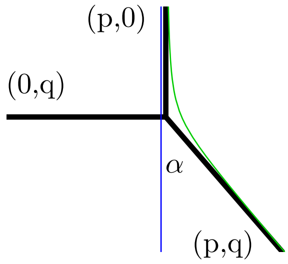


Figure 6: The string junction described in the text. The blue line represents the D1-D5 string in the background, and the green line shows the bending of the full solution (4.2).

It's interesting to look at the large r behaviour:

$$\begin{aligned} x &\sim -\frac{p}{gq} r + C \\ F_{tx} &\sim \frac{gq}{p} \end{aligned}$$

This is precisely the solution, in the flat space approximation, of a (p, q) string impinging on the D1-D5 string in the background, reaching it at an angle α with $\tan \alpha = g \frac{q}{p}$ [32, 33]. This configuration would arise as the third leg of a three string junction consisting of p D-strings parallel to the D1-D5 string and q fundamental strings orthogonal to it, joining up at $x = C$, as shown in figure 6. The full solution shows the bending or 'kinkiness' [34] under the influence of the D1-D5 string in the background, moving the junction point off to infinity and leaving only one leg visible. Hence we can see this solution as the result of placing a (p, q) string junction in the D1-D5 background. For the general solutions (4.1) a similar interpretation holds, the only difference being that the (p, q) string leg ends on a D3-brane in these cases.

The solution extending the attractor solution in the near-horizon limit is obtained by moving the D3-brane to $r = 0$. The solution becomes

$$\begin{aligned} \tilde{r} &= \sqrt{\left| \frac{\psi - \psi_*}{\sin \psi \cos \psi} \right|} \\ \tilde{x} &= \sqrt{\left| \frac{\cot \psi}{\psi - \psi_*} \right|} \end{aligned}$$

5. Supersymmetry analysis

In this section we show that the BPS equations (2.20) can alternatively be derived from the requirement of supersymmetry. The full supergravity background preserves 8 real 'Poincaré' supercharges, while in the near horizon region there are an additional 8 real 'conformal' supercharges. We will show that the BPS D-brane solutions discussed above also display the phenomenon of supersymmetry enhancement: near $r = 0$, supersymmetry is enhanced from 4 Poincaré supersymmetries preserved by the generic solution to include an extra 4 conformal supersymmetries preserved by the attractor solution.

A supersymmetry of the background is preserved in the presence of a bosonic D-brane configuration if it can be compensated for by a κ-symmetry transformation [35, 26]. This can be expressed as a projection equation

$$(1-\Gamma_\kappa)\epsilon = 0 \tag{5.1}$$

where Γ_κ (satisfying $\mathrm{tr}\Gamma_\kappa = 0,\ \Gamma_\kappa^2 = 1$) is the operator entering in the κ-symmetry transformation rule on the D-brane and ϵ is a general Killing spinor of the background pulled back to the world-volume.

The Poincaré Killing spinors of the background can be written as (see appendix B for a derivation in our conventions)

$$\epsilon = (H_1H_5)^{-1/8}R(\psi,\theta,\phi)\epsilon_0 \tag{5.2}$$

where $R(\psi,\theta,\phi)$ is a rotation

$$R(\psi,\theta,\phi) = e^{\frac{\frac{\pi}{2}-\psi}{2}\Gamma^{\underline{\theta\phi}}\sigma^1}e^{\frac{\frac{\pi}{2}-\theta}{2}\Gamma^{\underline{\psi\phi}}\sigma^1}e^{\frac{\phi}{2}\Gamma^{\underline{\psi\theta}}\sigma^1}.$$

The spinor ϵ_0 is constant on the six-dimensional space, covariantly constant on $\mathcal{M}$ and satisfies the projection equations

$$\begin{aligned}\left(1+\Gamma^{\underline{tx}}\sigma^1\right)\epsilon_0 &= 0\\ \left(1-\Gamma^{\underline{r\psi\theta\phi}}\sigma^1\right)\epsilon_0 &= 0\end{aligned} \tag{5.3}$$

The operator Γ_κ entering in (5.1) depends on the embedding of the D3-brane in the background as well as on the worldvolume gauge field. As before, we take the worldvolume coordinates to be (t,x,θ,ϕ). Imposing SU(2) symmetry as discussed in section 2, the induced worldvolume metric on a static D3-brane is

$$\begin{aligned}d\hat{s}^2 &= -(H_1H_5)^{-1/2}dt^2 + \left[(H_1H_5)^{-1/2} + (H_1H_5)^{1/2}(r'^2+r^2\psi'^2)\right]dx^2\\ &\quad + r^2(H_1H_5)^{1/2}\sin^2\psi(d\theta^2+\sin^2\theta d\phi^2).\end{aligned}$$

where a prime denotes a derivative with respect to x. The form of the worldvolume gauge fields is restricted by fixing the (p,q) charge and was given in the expressions (2.8), (2.15). In terms of the vielbein for the above induced metric the gauge fields read

$$F = \frac{\Delta_1}{\sqrt{\Delta_1^2+\Delta_2^2+\Delta_3^2}}e^{\underline{\hat{t}}}\wedge e^{\underline{\hat{x}}} + \frac{\Delta_3}{\Delta_2}e^{\underline{\hat{\theta}}}\wedge e^{\underline{\hat{\phi}}}$$

We use an index convention where a hatted index denotes a pullback to the worldvolume and orthonormal frame indices are underlined. The κ-operator Γ_κ is then [35, 26]

$$\Gamma_\kappa = e^{-\Phi_0\Gamma^{\underline{\hat{t}\hat{x}}}\sigma^3-\Phi_1\Gamma^{\underline{\hat{\theta}\hat{\phi}}}\sigma^3}\Gamma_{\underline{\hat{t}\hat{x}\hat{\theta}\hat{\phi}}}i\sigma^2$$

with $\Phi_0,\ \Phi_1$ defined by

$$\tanh\Phi_0 = \frac{\Delta_1}{\sqrt{\Delta_1^2+\Delta_2^2+\Delta_3^2}};\qquad \tan\Phi_1 = \frac{\Delta_3}{\Delta_2}.$$

The pulled-back gamma matrices are related to the 10-dimensional ones as

$$\begin{aligned}\Gamma_{\underline{\hat{t}\hat{x}}} &= \frac{1}{\sqrt{1+(H_1H_5)(r'^2+r^2\psi'^2)}}\left(\Gamma_{\underline{tx}}+(H_1H_5)^{1/2}r'\Gamma_{\underline{tr}}+(H_1H_5)^{1/2}r\psi'\Gamma_{\underline{t\psi}}\right)\\ \Gamma_{\underline{\hat{\theta}\hat{\phi}}} &= \Gamma_{\underline{\theta\phi}}\end{aligned}$$

Requiring $\Gamma_\kappa\epsilon=\epsilon$ with ϵ given in (5.2) for all values of θ and ϕ leads to two equations which can be summarized as

$$\left(1-e^{s\Phi_0\Gamma_s\sigma^3}e^{-\Phi_1\Gamma^{\underline{\theta\phi}}\sigma^3}\Gamma_s\Gamma^{\underline{\theta\phi}}i\sigma^2\right)e^{s\frac{\frac{\pi}{2}-\psi}{2}\Gamma^{\underline{\theta\phi}}\sigma^1}\epsilon_0=0 \tag{5.4}$$

where we defined the operator Γ_s

$$\Gamma_s=\frac{1}{\sqrt{1+H_1H_5(r'^2+r^2\psi'^2)}}\left(\Gamma_{\underline{tx}}+(H_1H_5)^{1/2}r'\Gamma_{\underline{tr}}+s(H_1H_5)^{1/2}r\psi'\Gamma_{\underline{t\psi}}\right)$$

and s can be 1 or -1. Some algebraic manipulations reduce the equations (5.4) to the following system

$$\begin{aligned}(H_1H_5)^{1/2}\Delta_3 r\psi' &= \pm[\cos\psi\Delta_2-\sin\psi\Delta_1]\\ (H_1H_5)^{1/2}\Delta_3 r' &= \pm[\sin\psi\Delta_2+\cos\psi\Delta_1]\\ (1\pm\Gamma_{\underline{t\psi}}\sigma^3)\epsilon_0 &= 0\end{aligned} \tag{5.5}$$

The first two equations are identical to (2.20), as one can see by making use of

$$\begin{aligned}\partial_r(rZ) &= \sin\psi\Delta_2+\cos\psi\Delta_1\\ \partial_\psi Z &= \cos\psi\Delta_2-\sin\psi\Delta_1\end{aligned}$$

The projector in (5.5) commutes with (5.3), showing that the solutions preserve 4 out of the 8 Poincaré supersymmetries. Note that the solutions with different sign choices in (5.5) are not mutually BPS.

We now proceed to verify whether, in the near horizon limit $a=0$, any of the solutions preserve some of the enhanced conformal supersymmetries. These are given by (see appendix B for details):

$$\tilde{\epsilon}=\left(\frac{1}{\sqrt{u}}+\sqrt{u}(t\Gamma^{\underline{tu}}-x\Gamma^{\underline{xu}})\right)R(\psi,\theta,\phi)\tilde{\epsilon}_0 \tag{5.6}$$

where we defined a rescaled radial coordinate $u\equiv\frac{r}{r_1r_5}$. The spinor $\tilde{\epsilon}_0$ is constant on $AdS_3\times S^3$, covariantly constant on $\mathcal{M}$ and satisfies the projection equations

$$\begin{aligned}\left(1-\Gamma^{\underline{tx}}\sigma^1\right)\tilde{\epsilon}_0 &= 0\\ \left(1+\Gamma^{\underline{r\psi\theta\phi}}\sigma^1\right)\tilde{\epsilon}_0 &= 0.\end{aligned} \tag{5.7}$$

We look for solutions of $(1-\Gamma_\kappa)\tilde{\epsilon}=0$, where Γ_κ is as in (5.1) with a put to zero in the harmonic functions. In particular, the pulled-back gamma matrices are

$$\begin{aligned}\Gamma_{\underline{\hat{t}\hat{x}}} &= \frac{1}{\sqrt{1+\frac{1}{u^4}(u'^2+u^2\psi'^2)}}\left(\Gamma_{\underline{tx}}+\frac{u'}{u^2}\Gamma_{\underline{tu}}+\frac{1}{u}\psi'\Gamma_{\underline{t\psi}}\right)\\ \Gamma_{\underline{\hat{\theta}\hat{\phi}}} &= \Gamma_{\underline{\theta\phi}}\end{aligned}$$

Since the background Killing spinor $\tilde{\epsilon}$ is time dependent and we are looking for static solutions of $(1-\Gamma_\kappa)\tilde{\epsilon}=0$, the only possibility is for the coefficient of t in this equation to vanish separately. This leads to two equations

$$\begin{aligned} (1-\Gamma_\kappa)\Gamma^{\underline{tu}}R(\psi,\theta,\phi)\tilde{\epsilon}_0 &= 0 \qquad (5.8)\\ (1-\Gamma_\kappa)(\Gamma^{\underline{tu}}-ux\Gamma^{\underline{tx}})\Gamma^{\underline{tu}}R(\psi,\theta,\phi)\tilde{\epsilon}_0 &= 0 \qquad (5.9)\end{aligned}$$

Making use of the properties of $\tilde{\epsilon}_0$ in (5.7), one can show that the first equation leads to the equations we had before in (5.5), with ϵ_0 replaced by $\tilde{\epsilon}_0$. If these are satisfied, the second equation becomes equivalent to

$$[\Gamma_{\underline{\hat{t}\hat{x}}},(\Gamma_{\underline{tu}}-ux\Gamma_{\underline{tx}})]\Gamma^{\underline{tu}}R(\psi,\theta,\phi)\tilde{\epsilon}_0=0. \qquad (5.10)$$

The commutator equals

$$[\Gamma_{\underline{\hat{t}\hat{x}}},(\Gamma_{\underline{tu}}-ux\Gamma_{\underline{tx}})]=\frac{2}{\sqrt{1+\frac{1}{u^4}(u'^2+u^2\psi'^2)}}\left((1+\frac{xu'}{u})\Gamma_{\underline{xu}}+\frac{\psi'}{u}\Gamma_{\underline{\psi u}}-x\psi'\Gamma_{\underline{\psi x}}\right).$$

The equation (5.10) requires $\det[\Gamma_{\underline{\hat{t}\hat{x}}},(\Gamma_{\underline{tu}}-ux\Gamma_{\underline{tx}})]^2=0$ which leads to

$$(1+\frac{xu'}{u})^2+\left(\frac{\psi'}{u}\right)^2+(x\psi')^2=0.$$

Hence we see that the solutions preserving conformal supersymmetries have to satisfy

$$\begin{aligned} 1+\frac{xu'}{u} &= 0\\ \psi' &= 0.\end{aligned}$$

This singles out the attractor solution found in (3.6). It preserves 4 extra conformal supersymmetries specified by the projection

$$(1\pm\Gamma_{\underline{t\psi}}\sigma^3)\tilde{\epsilon}_0=0.$$

The supersymmetry preservation of the attractor solution was studied before in [14].

6. S-dual solutions

The solutions of in sections 3), (4 can of course be transformed to solutions in different duality frames for the background 2-charge system. Of special interest is the S-dual F1-NS5 background composed of fundamental strings and Neveu-Schwarz fivebranes. This background can, in the near-horizon region, be described as an exact conformal field theory on the SL$(2,R)\times$SU(2) Wess-Zumino-Witten model at level Q_5. We will now describe how our solutions transform under S-duality.

The background geometry transforms into

$$
\begin{aligned}
e^{-\Phi'} &= e^{\Phi} = \frac{1}{g'}\left(\frac{H_1}{H_5}\right)^{1/2}; \qquad g' = 1/g \\
ds'^2 &= e^{-\Phi} ds^2 = g'\left[(H_1)^{-1}(-dt^2+dx^2) + H_5(dr^2 + r^2 d\Omega_3^2) + \left(\frac{H_1}{H_5}\right) ds^2_{\mathcal{M}}\right] \\
B'^{(2)} &= C^{(2)} = \frac{g'}{H_1} dt\wedge dx + g' r_5^2(\psi - \sin\psi\cos\psi)\sin\theta d\theta\wedge d\phi
\end{aligned}
\tag{6.1}
$$

The transformed D3-brane solutions have an induced metric given by $d\hat{s}'^2 = e^{-\Phi} d\hat{s}^2$, while the equations of motion and the Bianchi identities for the worldvolume gauge field are reversed [38 – 40]:

$$
\begin{aligned}
F' &= \star K \\
\star K' &= -F
\end{aligned}
\tag{6.2}
$$

where K is defined as

$$
K^{\mu\nu} \equiv \frac{1}{\mu_3\sqrt{-\det \hat{G}}}\frac{\delta S}{\delta F_{\mu\nu}}.
$$

The charge quantization conditions (2.5), (2.6) then imply

$$
\begin{aligned}
q &= \frac{\mu_3}{\mu_1}\int_{S^2} F' \\
p &= -\frac{\mu_3}{\mu_1}\int_{S^2} \star K'.
\end{aligned}
$$

These are the usual quantization conditions for a $(D1, F1) = (q, p)$ string in the F1-NS5 background [8, 41]. The fact that, in the near-horizon limit, q is a $\mathbf{Z}_{Q_5}$-valued charge is also well-established in this case [42 – 45]. Applying the transformation (6.2) one finds

$$
\begin{aligned}
F'_{\theta\phi} &= -\frac{q\mu_1}{4\pi\mu_3}\sin\theta \\
F'_{tx} &= 0
\end{aligned}
$$

Defining $L'^2 = Q_5\alpha'$, the induced metric and the gauge-invariant field strength $\mathcal{F} = F + B$ become

$$
\begin{aligned}
d\hat{s}^2 &= \frac{g'}{H_1}\left(-dt^2 + \frac{\Delta_1^2+\Delta_2^2+\Delta_3^2}{\Delta_3^2}dx^2\right) + L'^2\Delta_2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \\
\mathcal{F} &= \frac{g'}{H_1} dt\wedge dx - L'^2\Delta_1\sin\theta d\theta\wedge d\phi.
\end{aligned}
\tag{6.3}
$$

For the open string metric $g^o_{\mu\nu} = g_{\mu\nu} - \mathcal{F}_{\mu\rho}g^{\rho\sigma}\mathcal{F}_{\sigma\nu}$, one finds:

$$
ds_o^2 = \frac{g'(\Delta_1^2+\Delta_2^2)}{H_1}\left(-\frac{dt^2}{\Delta_1^2+\Delta_2^2+\Delta_3^2} + \frac{dx^2}{\Delta_3^2}\right) + \frac{L'^2(\Delta_1^2+\Delta_2^2)}{\Delta_2}\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{6.4}
$$

To evaluate these formulas, one has to use the definitions (2.14) together with the solutions (3.10) or (4.1). For example, one easily checks that, in the near-horizon limit, the attractor solution is $AdS_2 \times S^2$ with radii in the induced metric given by given by

$$R_{\mathrm{AdS}_2} = L' \frac{T'^{\mathrm{attr}}_{(q,p)}}{T'^{\mathrm{attr}}_{(q,0)}}; \qquad R_{\mathrm{S}^2} = L' Z_* \tag{6.5}$$

where we defined

$$T'^{\mathrm{attr}}_{(q,p)} \equiv \mu_1 \sqrt{p^2 + (\frac{Q_5}{\pi} \sin \frac{p\pi}{Q_5})^2 e^{-2\Phi'}}.$$

In the open string metric (2.25), the radii become equal to the radius of the background geometry:

$$R^o_{\mathrm{AdS}_2} = R^o_{\mathrm{S}_2} = L'. \tag{6.6}$$

This particular solution was first studied in [15] and the above agrees with the results obtained there.

7. Discussion

In this paper we have discussed a simple example of a supersymmetric attractor mechanism in an open string setting. We found many similarities with the well-known attractor mechanism for supersymmetric black holes. This raises quite a few questions to which we have not given a satisfactory answer.

An obvious (but perhaps naive) question is whether, like in the case of black holes, the physics underlying the attractor mechanism is related to a microscopic entropy carried by open string degrees of freedom. A better understanding of the connection between the open string attractor mechanism and the open string version of the OSV conjecture, proposed in [9] in a different setting, could shed light on this issue.

Another question concerns the generalization of the mechanism to other backgrounds and the identification of the general conditions under which it occurs. As in the closed string case, one would expect a close relation to the mechanism for open string moduli stabilization which was discussed in [7]. As remarked in the Introduction, one would also not expect the mechanism to be restricted to supersymmetric cases.

For black holes, special geometry plays an important role in the attractor mechanism. It would be useful to gain more insight in the supersymmetric geometry underlying the attractor mechanism in the present example. This would require a better understanding of the how the superconformal symmetry algebra of the background gets realized nonlinearly on the D3-brane worldvolume. One of the quantities for which we would like to have a better interpretation is the function which we called Z and which controls the attractor flow.

After S-dualizing, we found brane solutions in the near-horizon limit of the F1-NS5 background. These should be describable as boundary states which cannot be obtained by tensoring together $\mathrm{SL}(2,R)$ and $\mathrm{SU}(2)$ boundary states, and it would be interesting to obtain these.

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And finally, our attractor flow solutions in the near-horizon limit of the D1-D5 system merit an interpretation from the point of the dual CFT. All solutions run to the boundary of AdS_3, which they intersect in a line. Hence they should correspond to line defects in the CFT, generalizing the ones studied in [46]. Similar branes in the $AdS_5 \times S^5$ background were given a dual CFT interpretation as Wilson lines in an antisymmetric representation in [47, 48]. It would also be of interest to construct the 'bubbling' solutions incorporating backreaction.

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A. Description as fuzzy (p, q) strings

Our D3-brane configurations have, in a certain regime of the parameters, an equivalent description as (p, q) strings expanded to form a D3 brane on a fuzzy S^2. We will now describe this version of the Myers effect and show that, in the relevant regime, the solutions arise from the noncommutative worldvolume theory for p D-strings in the D1-D5 background.

We start by introducing auxiliary Cartesian coordinates Y^i, $i = 1, 2, 3$ satisfying the constraint $\sum_i (Y^i)^2 = 1$ such that the volume element on the S^2 can be written as

$$\sin\theta d\theta \wedge d\phi = \frac{1}{2}\epsilon_{ijk} Y^i dY^j \wedge dY^k.$$

We now consider Myers' action [25] for p D-strings in the given background. We choose a static gauge where the worldvolume is parametrized by t, x. The worldvolume fields now become $p \times p$ matrices. We will restrict attention to static configurations, starting from an ansatz of the form

$$\begin{aligned} F_{tx} &= F_{tx}(x)\mathbf{1}_{p\times p} \\ r &= r(x)\mathbf{1}_{p\times p} \\ \Psi &= \psi(x)\mathbf{1}_{p\times p}. \end{aligned}$$

Furthermore, we take Y^i to be arbitrary constant matrices satisfying the constraint $\sum_i (Y^i)^2 = 1$. The latter can be implemented by introducing a (matrix-valued) Lagrange multiplier λ. The multi-D1 brane action at leading order then takes the form [25]

$$\begin{aligned} S = -\mu_1 \mathrm{Tr} \int dt dx \Big[& e^{-\Phi}\sqrt{-\det(P[G_{ab}] + F_{ab})}\sqrt{\det Q^i_j} \\ & + C^{(2)}_{tx} + \frac{i}{2\pi\alpha'} i_Y i_Y C^{(2)} F_{tx} + \lambda(\sum_i (Y^i)^2 - 1)\Big] \end{aligned} \tag{A.1}$$

where

$$
\begin{aligned}
\sqrt{-\det(P[G_{ab}]+F_{ab})} &= \sqrt{(H_1H_5)^{-1}+r'^2+r^2\psi'^2-F_{tx}^2}\\
\sqrt{\det Q^i_j} &= 1-\frac{H_1H_5r^4\sin^4\psi}{4(2\pi\alpha')^2}\sum_{i,j}[Y^i,Y^j]^2\\
i_Yi_YC^{(2)} &= \frac{r_5^2}{2g}(\psi-\sin\psi\cos\psi)\epsilon_{ijk}Y^i[Y^j,Y^k]
\end{aligned}
$$

The equations of motion for the matrices Y following from this action are

$$
\begin{aligned}
0 =& \frac{1}{g}\sqrt{\frac{H_5}{H_1}}\sqrt{(H_1H_5)^{-1}+r'^2+r^2\psi'^2-F_{tx}^2}\frac{H_1H_5r^4\sin^4\psi}{(2\pi\alpha')^2}\sum_j Y^j[Y^i,Y^j]\\
&+\frac{3ir_5^2}{4\pi\alpha' g}(\psi-\sin\psi\cos\psi)\epsilon_{ijk}[Y^j,Y^k]F_{tx}+2\lambda Y^i
\end{aligned}
\tag{A.2}
$$

When the Y's are taken to form a fuzzy two-sphere, $[Y^i,Y^j]\sim i\epsilon_{ijk}Y^k$, one sees that each term in this equation is proportional to Y^i. Hence the equation is trivially solved by adjusting the Lagrange multiplier. In other words, the variation of the action around a fuzzy sphere configuration is proportional to $\sum Y^i\delta Y^i$ and vanishes for variations on the constraint surface. The necessary ingredients that went into this are the fact that the auxiliary Y^i-space is flat, and that the background magnetic potential $C^{(2)}_{\rm magn}$ is constant over the S^2.

In terms of matrices t^i in the p-dimensional irreducible representation of SU(2), satisfying $[t^i,t^j]=-i\epsilon_{ijk}t^k$, the Y's are

$$
Y^i=\frac{2}{\sqrt{p^2-1}}t^i.
$$

Substituting into the action (A.1) leads to

$$
\begin{aligned}
S =& -p\mu_1\int dtdx\Bigg[\frac{1}{g}\sqrt{\frac{H_5}{H_1}}\sqrt{(H_1H_5)^{-1}+r'^2+r^2\psi'^2-F_{tx}^2}\left(1+\frac{2H_1H_5r^4\sin^4\psi}{(p^2-1)(2\pi\alpha')^2}\right)\\
&+\frac{r_1^2}{gH^1}+\frac{Q_5}{\sqrt{p^2-1}\pi}(\psi-\sin\psi\cos\psi)F_{tx}\Bigg].
\end{aligned}
$$

We see that this expression agrees with (2.9) for static configurations in the large p limit when

$$
\frac{H_1H_5r^4\sin^4\psi}{p^2(\alpha'\pi)^2}\ll 1.
$$

The equations (A.2) also allow for a solution where the the Y^i are commuting, forcing the Lagrange multiplier λ to be zero. This represents a (p,q) string which has not expanded into a fuzzy 2-sphere. The existence of solutions for both expanded and non-expanded configurations is similar to the case of giant gravitons [28, 29].

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B. Killing spinors

In this appendix we derive the form of the Killing spinors in the D1-D5 background. We follow the conventions of [49], in which the type IIB gravitino and dilatino variations (for vanishing $H,\ F^{(1)}, F^{(5)}$) are given by

$$\begin{aligned}\delta\lambda &= \left[\frac{1}{2}\Gamma^M\partial_M - \frac{1}{4}e^{\Phi}\not{F}^{(3)}\sigma^1\right]\epsilon \\ \delta\Psi_M &= \left[\nabla_M + \frac{1}{8}e^{\Phi}\not{F}^{(3)}\Gamma_M\sigma^1\right]\epsilon.\end{aligned}$$

Here, ϵ is a doublet of chiral spinors in 10 dimensions with chirality $\Gamma_{(11)}\epsilon \equiv \Gamma^{\underline{0}\cdots\underline{9}}\epsilon = -\epsilon$.

The vanishing of the dilatino variation in the background (2.1) imposes the following projection equations:

$$\begin{aligned}\left(1+\Gamma^{\underline{tx}}\sigma^1\right)\epsilon &= 0 \\ \left(1-\Gamma^{\underline{r\psi\theta\phi}}\sigma^1\right)\epsilon &= 0\end{aligned}$$

where we use underlined indices to denote orthonormal frame indices. This projects the number of independent real components of ϵ down to 8. The near-horizon background $a=0$ allows extra solutions which give rise to 8 enhanced supersymmetries and which will be discussed below.

The vanishing of the gravitino variation determines the coordinate dependence of ϵ. The components on the internal manifold $\mathcal{M}$ lead to the condition the ϵ is covariantly constant with respect to the Ricci-flat metric on $\mathcal{M}$. The 6-dimensional components have the solution

$$\epsilon = (H_1H_5)^{-1/8}R(\psi,\theta,\phi)\epsilon_0 \tag{B.1}$$

where $R(\psi,\theta,\phi)$ is a rotation

$$R(\psi,\theta,\phi) = e^{\frac{\frac{\pi}{2}-\psi}{2}\Gamma^{\underline{\theta\phi}}\sigma^1}e^{\frac{\frac{\pi}{2}-\theta}{2}\Gamma^{\underline{\psi\phi}}\sigma^1}e^{\frac{\phi}{2}\Gamma^{\underline{\psi\theta}}\sigma^1}.$$

The spinor ϵ_0 is constant in 6 dimensions, covariantly constant on $\mathcal{M}$ and satisfies the projection equations $\Gamma^{\underline{tx}}\sigma^1\epsilon_0 = -\epsilon_0,\ \Gamma^{\underline{r\psi\theta\phi}}\sigma^1\epsilon_0 = \epsilon_0$.

In the near-horizon limit $a=0$, the dilatino equation allows extra solutions $\tilde{\epsilon}$ satisfying

$$\begin{aligned}\left(1-\Gamma^{\underline{tx}}\sigma^1\right)\tilde{\epsilon} &= 0 \\ \left(1+\Gamma^{\underline{r\psi\theta\phi}}\sigma^1\right)\tilde{\epsilon} &= 0\end{aligned}$$

The gravitino equations then lead to the following form of these enhanced supersymmetries:

$$\tilde{\epsilon} = \left(\sqrt{\frac{r_1r_5}{r}} + \sqrt{\frac{r}{r_1r_5}}\left(t\Gamma^{\underline{tr}} - x\Gamma^{\underline{xr}}\right)\right)R(\psi,\theta,\phi)\tilde{\epsilon}_0 \tag{B.2}$$

where $\tilde{\epsilon}_0$ is constant on $AdS_3\times S^3$, covariantly constant on $\mathcal{M}$ and satisfies the projection equations $\Gamma^{\underline{tx}}\sigma^1\tilde{\epsilon}_0 = \tilde{\epsilon}_0,\ \Gamma^{\underline{r\psi\theta\phi}}\sigma^1\tilde{\epsilon}_0 = -\tilde{\epsilon}_0$.

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